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LETTER TO THE EDITOR

Comment on the ghost problem in a higher derivative Yang–Mills theory

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Abstract. Consistency conditions have been proposed by Salam and Strathdee and by Julve and Tonin which, if satisfied, may make the tensor ghost in higher derivative gravitational theories innocuous. We show that this criterion cannot be satisfied in a pure Yang–Mills theory with higher derivatives.

New interest has been shown in the last few years in field theories with higher derivatives (Stelle 1977, Ferrara and Zumino 1977, Thirring 1978). This is due mainly to the fact that higher derivative terms improve the ultraviolet behaviour of the theory. Hence a quantum theory of gravity (DeWitt 1967, Stelle 1977) becomes renormalisable when terms like R^2, $R^{\mu\nu}R_{\mu\nu}$ are added to the Einstein Lagrangian. In another area, these theories may be of some relevance in quark confinement, since they can produce rising potentials (Thirring 1978). However, as is well known, such theories have unphysical negative norm states and a procedure must be found to make these harmless.

Recently, criteria have been proposed by Salam and Strathdee (1978) and independently by Julve and Tonin (1978) which, if satisfied, may make the metric ghost innocuous. These criteria are based upon renormalisation group considerations. The use of a running parameter instead of the renormalised mass may allow one to see if the ghost decouples in the limit of high energies, i.e. the effective mass $M(p)$ should go to infinity at least as fast as p when the latter tends to infinity. If this is indeed the case, it is possible that the pole in the ghost propagator

$$\frac{1}{p^2-M^2(p)}$$

is also shifted to infinity, thereby rendering the ghost harmless.

Following Lee (1974) and its notation, the effective mass and coupling constant are given by

$$M(k)=\frac{M_R}{k}\exp\left(-\int_1^k H_M(g(k'))\frac{\mathrm{d}k'}{k'}\right) \tag{1}$$

$$k\frac{\mathrm{d}g(k)}{\mathrm{d}k}=\beta(g(k)) \tag{2}$$

where H_M and $\beta(g)$ are the renormalisation group functions associated with M and g.

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In perturbation theory $H_m(g(k)) \sim g^2(k) + \ldots$ and we see that the criterion is, in fact, incompatible with asymptotic freedom, for in this case $g^2(k)$ (and by implication, $M^2(k)$) would tend to zero as k goes to infinity. Instead $g^2(k)$ should tend towards a constant when $k \to \infty$, which means that the β function should have a non-Gaussian fixed point. The precise statement of this criterion is therefore non-perturbative. What one can check in perturbation theory, however, is whether the theory is asymptotically free or not.

We consider a pure Yang–Mills field, with higher derivatives. Taking into account the Bianchi identity, the most general Lagrangian is

$$\mathscr{L} = -\tfrac{1}{2}\operatorname{Tr} F_{\mu\nu}F^{\mu\nu} + \frac{1}{M^2}\operatorname{Tr}(D_\rho F^{\rho\nu} D^\sigma F_{\sigma\nu}) - \frac{2\mathrm{i}}{3}\frac{\gamma g}{M^2}\operatorname{Tr}(F_{\mu\nu}F^{\nu\rho}F_{\rho\mu}) \tag{3}$$

where g is the gauge coupling and γ is a real arbitrary coupling parameter, and we use the following conventions:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g[A_\mu, A_\nu]$$
$$D_\rho F_{\mu\nu} = \partial_\rho F_{\mu\nu} + \mathrm{i}g[F_{\mu\nu}, A_\rho]$$
$$A_\mu = A^a_\mu t^a$$
$$[t^a, t^b] = \mathrm{i}C_{abc}t^c$$

with C_{abc} being totally antisymmetric,

$$\operatorname{Tr} t^a t^b = \tfrac{1}{2}\delta_{ab}.$$

To this Lagrangian, we add the usual gauge fixing and Faddeev–Popov terms:

$$-\frac{1}{\alpha}\operatorname{Tr}(\partial_\mu A^\mu)^2 + 2\operatorname{Tr}\{\partial_\mu\phi^*\partial_\mu\phi - \mathrm{i}g\,\partial_\mu\phi^*[A_\mu, \phi]\}.$$

The propagator of this theory is given by

$$G^{ab}_{\mu\nu} = \delta^{ab}\left(M^2 g_{\mu\nu} - [M^2 + \alpha(p^2 - M^2)]\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2(p^2 - M^2)}.$$

We shall work in the transverse gauge, $\alpha = 0$, in which the inverse propagator has a high-energy behaviour of p^4.

One observes that the theory has a massive vector ghost in addition to the usual massless vector boson.

By power counting, it is easy to see that the only necessary counter-term is of the form (Slavnov 1971)

$$-\tfrac{1}{2}(Z-1)\operatorname{Tr} F_{\mu\nu}F^{\mu\nu}.$$

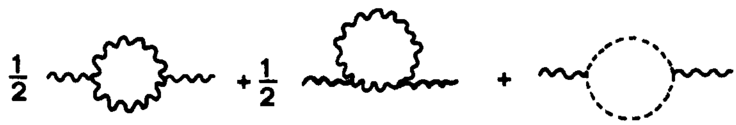
The bare charge and bare ghost mass are given by

$$g_{\text{bare}} = Z^{-1/2} g_R$$

and

$$M_{\text{bare}} = Z^{1/2} M_R.$$

The renormalisation constant Z is most easily determined from the two-point functions. The relevant graphs are



where the wavy line is a gauge boson and the broken line is the Faddeev–Popov ghost. The symbolic manipulation programme 'Schoonschip' (Strubbe 1974) has been used to handle the extensive algebra involved. The result is

$$Z = 1 + \frac{g^2 C_2(G)}{16\pi^2}\,\frac{3\gamma^2+36\gamma+43}{3}\,\frac{1}{4-n}$$

where

$$\sum_{cd} C_{acd}C_{bcd} = C_2(G)\delta_{ab}.$$

From this we have, following Lee (1974),

$$\beta(g) = \frac{1}{2}g^2\frac{\partial R}{\partial g} \qquad H_M(g) = \frac{1}{2}g\frac{\partial R}{\partial g}$$

where, to lowest order in perturbation theory,

$$R = -\frac{C_2(G)}{16\pi^2}\,\frac{3\gamma^2+36\gamma+43}{6}g^2+\ldots.$$

The function $3\gamma^2+36\gamma+43$ has two roots at $\gamma \simeq -1{\cdot}4$ and $-10{\cdot}6$, for which values the theory is one-loop-finite. For $-10{\cdot}6<\gamma<-1{\cdot}4$ the theory is infrared free and is asymptotically free outside this range.

Although it is possible to preclude the theory from being asymptotically free by an appropriate choice of γ, the Salam–Strathdee and Julve–Tonin criterion cannot be satisfied. Indeed, as a consequence of the super-renormalisability of the theory, one has the relation

$$\beta = gH_m \tag{4}$$

from which it is easily deduced that

$$\frac{M(k)}{M_R} = \frac{1}{kg(k)}. \tag{5}$$

If the β function has a non-Gaussian fixed point, $g(k)$ tends to a non-vanishing constant as k goes to infinity; hence $M(k)$ tends to zero. On the other hand, if there is no fixed point, $g(k)$ increases without bound and $M(k)$ tends to zero even faster. This asymptotic behaviour of $M(k)$ is a consequence of relation (4), here reflecting the super-renormalisability of the theory.

In a supersymmetric version of higher derivative gravity, such relations may appear due to presumably improved ultraviolet behaviour, and the favourable situation of the Julve–Tonin calculation may be lost.

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